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DETERMINATION OF GALVANOMAGNETIC AND THERMOMAGNETIC EFFECTS IN SEMICONDUCTORS

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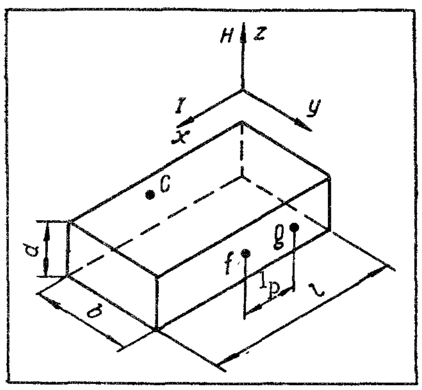
The factors with an appreciable influence on the accuracy of determination of the basic galvanomagnetic and thermomagnetic effects in semiconductors are examined. Formulas are given for the Hall emf, magneto resistance, and transverse Nernst-Ettingshausen effect with allowance for the nonisothermicity of the specimens and heat transfer from their surface to the surrounding medium.

The main difficulty in investigating galvano- and thermomagnetic effects arises from the need to exclude different kinds of parasitic emf's, including secondary galvano- and thermomagnetic effects superimposed on the basic measured quantity [1-4].

It is of considerable interest to examine and to evaluate quantitatively the influence of various factors on the accuracy of measurement of galvano- and thermomagnetic phenomena (Hall effect, magneto resistance, and transverse Nernst-Ettingshausen effect) under the actual experimental conditions. We begin with an analysis of the Hall effect.

Let the specimen be a parallelepiped located in a magnetic field; the current and the field are oriented as shown in the figure. The emf is measured by probes located on opposite faces of the specimen at right angles to the edge b. Here and below we consider, for simplicity, only isotropic semiconductors with a simple band structure and carriers of one sign, specifically electrons with charge e, and assume the field to be weak ($un/c \ll 1$).



Orientation of electric current and magnetic field and location of measuring probes in test specimen.

1. The temperature is the same at all point of the specimen. Then the isothermal Hall emf is given by the expression [2, 5]:

$$(E_x)_i = (R_x)_i \, IH/d, \tag{1}$$

where

$$(R_x)_i = A/enc.$$

The coefficient A is determined by the diffusion mechanism and the degree of degeneracy of the electron gas.

2. The temperature at the ends of the specimen $T_a = T_b =$ const.

a) There is no heat transfer from the lateral surfaces of the specimen (adiabatic conditions). Here the Ettingshausen effect – temperature gradient perpendicular to magnetic field and current – is also present at the same time as the Hall effect. If the coefficient of thermal emf, α, of the test material is not zero, this temperature drop produces an Ettingshausen emf E_e superimposed on the Hall emf and given by

$$E_e = \alpha b \left(\frac{dT}{dy} \right)_e = B(r, \mu^*) \frac{k\alpha}{e} \frac{\sigma}{\lambda} T R_x \frac{I_x H}{d}, \tag{2}$$

where

$$B(r, \mu^*) = \frac{2r + 3/2}{2r + 1/2} \frac{F_{2r+1/2}(\mu^*)}{F_{2r-1/2}(\mu^*)} - \frac{r+2}{r+1} \frac{F_{r+1}(\mu^*)}{F_r(\mu^*)}.$$

For a nondegenerate semiconductor [2] $B(r, \mu^*) = r - 1/2$, and for the degenerate case $B(r, \mu^*) = (2r - 1)/(2r + 2)$. Thus the total emf between the Hall probes is

$$E_{cf} = (E_x)_i \left[1 + B(r, \mu^*) \frac{k}{e\alpha} \mathrm{Io} \right]. \tag{3}$$

b) Heat transfer from the lateral surfaces of the specimen is not zero.

In order to estimate the influence of heat transfer on the temperature drop ΔT_e due to the Ettingshausen effect, we shall examine the thermal conditions in the specimen shown in the figure. We assume that there is no temperature gradient in the x and z directions. This condition is satisfied well enough at small Biot numbers $ab/\lambda < 1$ when the specimen length $l > 2b$. For steady conditions the heat conduction equation and the boundary conditions, taking into account Ettingshausen heat and heat transfer to the surrounding medium, have the form

$$\frac{d^2T'}{dx^2} - \frac{2a}{\lambda d} T' + \frac{j^2}{\sigma\lambda} = 0, \tag{4}$$

$$w_e + \lambda \frac{dT'(0)}{dy} - aT'(0) = 0, \tag{5}$$

$$w_e + \lambda \frac{dT'(b)}{dy} - aT'(b) = 0, \tag{6}$$

$$w_e = B(r, \mu^*) \frac{k}{e} \sigma T R_x jH. \tag{7}$$

Solving (4) with conditions (5) and (6), we obtain the temperature drop in the y direction:

$$\Delta T = \frac{wb}{\lambda}(1 - \zeta), \tag{8}$$

$$\zeta = \frac{2ab^3}{\lambda d(b+d)}\left[1 - \frac{2}{3}\frac{d}{b} + \frac{1}{4}\left(\frac{d}{b}\right)^2\right].$$

The quantity ζ is a correction allowing for heat transfer from the surface of the specimen. In this case the Hall emf measured between the probes may be written

$$E_{cf} = (E_x)_i\left[1 + B(r, \mu^*)\frac{k}{e\alpha} \mathrm{Jo}(1 - \zeta)\right]. \tag{9}$$

Here the Joffe number Jo is based on the mean specimen temperature T_m.

Thus, it follows from (9) that the heat transfer from the specimen surface reduces the Ettingshausen effect and correspondingly the nonisothermal part of the Hall emf. To determine the order of magnitude of the correction ζ, we shall consider, for example, what its value must be for a specimen with a ratio of thickness to width $d/b = 0.5$ and thermal conductivity $\lambda = 1.3$ W/m·deg at room temperature. Then $\zeta = 0.06$ in a vacuum ($a = 4.2$ W/m^2·deg) and $\zeta = 0.33$ in a chamber of still air ($a = 21$ W/m^2·deg). It is clear from this example that heat transfer leads to a reduced Ettingshausen effect and should be taken into account in measuring the latter. At the same time, it does not reduce the effect so much (at small Bi) that it may be neglected in measuring the Hall emf.

3. There is a longitudinal temperature gradient ($dT/dx \neq 0$) due to absorption and emission of Peltier heat at the ends of the specimen when a constant current I passes through it.

In a magnetic field this temperature drop in the specimen produces a transverse Nernst-Ettingshausen emf which is added to the Hall emf. The former is given by [5]

$$E_{\text{N-E}} = B(r, \mu^*)\frac{k}{e} \sigma R_x Hb \frac{dT}{dx}. \tag{10}$$

To determine the value of the temperature gradient dT/dx, we shall use results from [6], which gives an expression for the temperature distribution along the length of the specimen due to the Peltier effect, with allowance for heat transfer from the ends and sides of the specimen and heat losses from the lateral and thermocouple leads. For the central part of the specimen, where the Hall probes are located it is easy to show that

$$\frac{dT(l/2)}{dx} = \frac{\alpha j T_m}{\lambda(1 + \gamma')}, \tag{11}$$

where

$$\gamma' = \frac{ab}{2\lambda} + \frac{1}{6}\frac{a}{\lambda}\frac{b^2(b+d)}{ld} + \frac{b}{2\lambda dl}\sum^{n}\sqrt{ap_n\lambda_n S_n}. \tag{12}$$

Substituting (11) into (10), we find that the Nernst-Ettingshausen emf is

$$E_{\text{N-E}} = B(r, \mu^*)\frac{k}{e\alpha}\frac{\mathrm{Jo}}{(1+\gamma')}(E_x)_i. \tag{13}$$

Thus, taking (9) and (13) into account, the final formula for the Hall emf under actual experimental conditions is

$$E_{cf} = (E_x)_i\left[1 + B(r, \mu^*)\frac{k}{e\alpha} \mathrm{Jo}\left(\frac{1}{1+\zeta} + \frac{1}{1+\gamma'}\right)\right]. \tag{14}$$

It follows from (14) that the influence of the Ettingshausen and Nernst-Ettingshausen parasitic effects on the accuracy of determination of the Hall emf will be the greater, the larger the Joffe numbers of the test material, and at sufficiently large Jo they may distort the results considerably. For example, in the case of an atomic semiconductor with scattering at acoustic lattice vibrations ($r = 0$) and $\alpha = 170$-180 μV/deg and Jo = 0.6, the error in determining the Hall emf is 20-25%, allowing for heat transfer.

Let us now consider the influence on the accuracy of determination of the relative change in resistance ($\Delta\rho/\rho$) in a magnetic field of the longitudinal Nernst-Ettingshausen effect created by the temperature drop due to the Peltier effect. In the absence of a magnetic field, when the current is switched on, a potential difference $E_{\rho 0}$ is set up between the measuring probes f and g, equal to the sum of the ohmic voltage drop E_ρ^0 and the thermal emf E_{te} due to the Peltier effect. When the magnetic field is turned on, additional emf's $E_{\Delta\rho}$ and $E_{N\text{-}E}^l$ appear between the measuring probes; these are due respectively to change in specimen resistance and to the longitudinal Nernst-Ettingshausen effect: $E_\rho = E_\rho^0 + E_{te} + E_{\Delta\rho} + E_{N\text{-}E}^l$.

It is easy to see that the magnetoresistance equation, with account for the Peltier and Nernst-Ettingshausen effects, has the form

$$\left(\frac{\Delta\rho}{\rho_0}\right)_i = \left(\frac{\Delta\rho}{\rho}\right)_{\text{meas}}\left[1 + \frac{E_{te}}{E_\rho^{(0)}}\right]\left[1 + \frac{E_{N\text{-}E}^l}{E_{\Delta\rho}}\right]^{-1}. \tag{15}$$

Let us transform this expression by expanding the quantities entering into it. According to (11), the thermal emf due to the Peltier effect may be written as $E_{te} = j\alpha^2 T_m l_p/\lambda(1+\gamma')$.

The ohmic voltage drop is $E_\rho^{(0)} = j\rho_0 l_p$. Further, in a weak magnetic field, the longitudinal Nernst-Ettingshausen emf and the relative change in resistance are given [7] by

$$E_{N\text{-}E}^l = C(r, \mu^*)\,\frac{k}{e}\left(\frac{uH}{c}\right)^2 \Delta T_x,$$

$$\Delta\rho/\rho = D(r, \mu^*)\,(uH/c)^2.$$

Substituting the values of E_{te}, $E_{\rho 0}$, $E_{N\text{-}E}^l$ and $E_{\Delta\rho}$ into (15), we obtain a final expression for the change in relative resistance in a magnetic field:

$$\left(\frac{\Delta\rho}{\rho_0}\right)_i = \left(\frac{\Delta\rho}{\rho}\right)_{\text{meas}}\left[1 + \frac{\text{Jo}}{1+\gamma'}\right]\left[1 + \frac{C(r,\mu^*)}{D(r,\mu^*)}\,\frac{k}{e\alpha}\,\frac{\text{Jo}}{(1+\gamma')}\right]^{-1}. \tag{16}$$

Thus, in determining the latter it is necessary to take into account the fact that the measured value $(\Delta\rho/\rho_0)_{\text{meas}}$ differs from the true value by a certain quantity in the square brackets. This correction depends on the nature of the specimen and the experimental conditions, and becomes appreciable in measuring $\Delta\rho/\rho_0$ for thermoelectric materials.

We note that the Nernst and Maggi-Righi-Leduc effects, which have not been considered, may make the correction even greater.

Transverse Nernst-Ettingshausen effect. When a specimen with a temperature drop is placed in a magnetic field, two transverse thermomagnetic effects are observed: the Nernst-Ettingshausen effect (an emf $E_{N\text{-}E}$ in a direction perpendicular to the temperature gradient and the magnetic field) and the Righi-Leduc effect (a temperature drop ΔT_y in the same direction producing a thermal emf $E_{R\text{-}L}$ that is added to $E_{N\text{-}E}$). Thus the total emf in the y direction is

$$(E_{N\text{-}E})_{\text{meas}} = (E_{N\text{-}E})_i + E_{R\text{-}L}. \tag{17}$$

The Righi-Leduc emf is given [7] by the expression

$$E_{R\text{-}L} = G(r, \mu^*)\alpha\sigma R_x H b\,\frac{\lambda_e}{\lambda}\,\frac{dT}{dx}. \tag{18}$$

The coefficients $G(r, \mu^*)$ for nondegenerate and degenerate semiconductors, respectively, are

$$G_{nd}(r, \mu^*) = \frac{(r+1/2)^2 + 3/2}{(r+2)},\quad G_d\,(r, \mu^*) = 1.$$

Or, finally, taking into account heat transfer from the specimen surface as well, we have

$$(E_{N\text{-}E})_{\text{meas}} = (E_{N\text{-}E})_i\left[1 + \frac{G(r,\mu^*)}{B(r,\mu^*)}\,\frac{\alpha e}{k}\,\frac{\lambda_e}{\lambda}(1-\zeta)\right]. \tag{19}$$

This formula may be transformed by replacing the electronic component of thermal conductivity with values based on the Wiedemann-Franz law $\lambda_{e_{nd}} = (r+2)\left(\frac{k}{e}\right)^2 \sigma T$ and $\lambda_{ed} = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2 \sigma T$.

Then, allowing for heat transfer, we have for the nondegenerate semiconductor

$$(E_{N-E})_{meas} = (E_{N-E})_i \left[1 + \frac{(r+1/2)^2 + 3/2}{(r-1/2)} \frac{k}{e\alpha} Jo(1-\zeta) \right] \tag{20}$$

and for the degenerate case

$$(E_{N-E})_{meas} = (E_{N-E})_i \left[1 + \frac{\pi^2}{3} \frac{(2r+2)}{(2r-1)} \left(\frac{k}{e\alpha}\right)^2 Jo(1-\zeta) \right]. \tag{21}$$

We note that earlier, in evaluating the error in measuring the Hall emf, we neglected the influence of the Righi-Leduc effect, which may be created by the Peltier effect at the ends of the specimen. Taking this into account, and using (19), we can transform (19) to

$$(E_{cf})_{meas} = (E_x)_i \left\{ 1 + B(r, \mu^*) \frac{k}{e\alpha} Jo \left[\frac{1}{1+\zeta} + \frac{1}{1+\gamma'} \right] + G(r, \mu^*) \frac{\lambda_e(1-\zeta)}{\lambda(1+\gamma')} Jo \right\}. \tag{22}$$

The analysis thus allows one to estimate the influence of the nonisothermicity of the specimen and of heat transfer from its surfaces to the surrounding medium on the accuracy of determination of the basic galvanomagnetic and thermomagnetic phenomena. It follows from (16), (20), (21), and (22) that these factors become especially important in testing materials with thermoelectric properties.

NOTATION

l, b, d – linear dimensions of specimen; I, j, N, and T – current, current density, magnetic field intensity, and absolute temperature; α, λ, σ, and R_x – thermal emf, thermal conductivity, electric conductivity, and Hall constant of test material; n, u, and c – carrier concentration and mobility and speed of light; B(r, μ^*) – coefficient determined by the scattering mechanism and degree of degeneracy of the electron gas; k, r, μ^* – Boltzmann constant, exponent in the relation between the electron mean free path and energy, and reduced chemical potential level; F(μ^*) – Fermi integral; Jo = $a^2\sigma T/\lambda$ – Joffe number characterizing the energy potential of thermoelectric materials; T' = T(y)-T_0, where T_0 is the ambient temperature; a, ω_e – coefficient of heat transfer from specimen surface and Ettingshausen heat emitted or absorbed at the specimen faces perpendicular to the y axis; λ_n, P_n, S_n – thermal conductivity, perimeter and cross-sectional area of Hall probes and thermocouple leads; G(r, μ^*), D(r, μ^*) – coefficients depending on the carrier scattering mechanism and the position of the chemical potential level.

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